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2007 J. Phys. A: Math. Theor. 40 5193

(http://iopscience.iop.org/1751-8121/40/19/018)

J. Phys. A: Math. Theor. **40** (2007) 5193–5204 doi:10.1088/1751-8113/40/19/018

The two-dimensional derivative-coupling model revisited

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Received 15 September 2006
Published 24 April 2007
Online at stacks.iop.org/JPhysA/40/5193

Abstract
Using the operator approach we reexamine the two-dimensional model describing a massive Fermi field interacting via derivative couplings with two massless Bose fields, one scalar and the other pseudoscalar. Performing a canonical transformation on the Bose field algebra, the Fermi field operator is written in terms of the Mandelstam soliton operator and the derivative-coupling (DC) model is mapped into the massive Thirring model with two vector-current–scalar-derivative interactions (*Schroer–Thirring model*). The DC model with massless fermions can be mapped into the massless Rothe–Stamatescu model with a Thirring interaction (*massless Rothe–Stamatescu–Thirring model*). Within the present approach the weak equivalence between the fermionic sector of the DC model and the massive Thirring model is exhibited compactly.

PACS numbers: 11.15.Ex, 11.15.Tk

1. Introduction

In the past two-dimensional derivative coupling (DC) models have been the subjects of various investigations within different approaches [1–9]. The equivalence between the massless Thirring model and the DC model describing fermions interacting with two massless Bose fields, one scalar and the other pseudoscalar, via derivative couplings has been discussed in [6–8]. For a certain choice of the coupling parameters, the equivalence between the fermionic sector of the DC model and the Thirring model is established in a weak form between the fermionic Green's functions of the corresponding models. The weak equivalence only works under the expense of introducing opposite metric quantization for the bosonic fields [7, 8], or by considering one derivative-coupling term with an imaginary coupling parameter [6]. Under these assumptions the fermionic Green's functions of the DC model are mapped into those of the Thirring model obtained with Klaiber's and Johnson's solutions [10]. As a matter of fact, this is the only way under which the degrees of freedom in the two models can be artificially matched.

In [7] the connection between the two models is analysed within the operator formalism by comparing the corresponding operator solutions. In order to establish a correspondence between the operator solution of the massless Thirring model and the DC model, the Bose fields are considered with opposite metric and a combination of the original *three* bosonic degrees of freedom is introduced to define *two* new bosonic fields. After redefining the Bose fields, the operator solution is given in terms of both a 'spurious' field and the Thirring field operator. However, the artifice employed in [7] by introducing a Bose field redefinition to reduce the number of degrees of freedom is meaningless for the DC model with massive fermions, since it presupposes that the bosonic fields are free and massless. For massive fermions, the Bose field algebra contains two sine-Gordon-like pseudoscalar fields and a free massless scalar field. The field redefinition used in [7] mixes these three degrees of freedom to define the sine-Gordon soliton field and thus spoils the mass operator.

In [8] the DC model with massless fermions was analysed using the functional integral approach. The equivalence between the fermionic two-point functions of the DC model and the Thirring model is established with an appropriate relation between the coupling parameters of the two models. A one-to-one mapping between the operator solutions of the DC model and the massless Thirring model is established by imposing opposite metric quantization for the Bose fields and a relation between the coupling parameters of the two models. However, the equivalence established in [8] only works for massless fermions. In [6], the weak equivalence between the Thirring model and the DC model has been used to investigate the ultraviolet divergences and renormalizability in mass perturbation in the Thirring model.

Recently, in order to obtain a clear understanding of the actual role played by the fermionic quartic-self-interaction in the DC models, the model describing a massless pseudoscalar field interacting via *axial-current-pseudoscalar-derivative coupling* with massive fermions has been discussed in [11] using the operator approach. This model corresponds to the Rothe–Stamatescu model [2] in the zero mass limit for the pseudoscalar field when modified to include a mass term for the Fermi field (*modified Rothe–Stamatescu model* (*MRS model*)). It was shown that the presence of the Thirring interaction is an intrinsic property of the MRS model. The Thirring interaction is exhibited compactly by performing a canonical transformation on the Bose fields. The operator solution for the quantum equations of motion is written in terms of the Mandelstam Fermi field operator of the Thirring model interacting with a scalar field via *vector-current-derivative coupling* (*Schroer–Thirring model*). The vector current is mapped into the Thirring current, in such a way that the charge sectors of the MRS model are mapped into the charge sectors of the Thirring model. In this way, the bosonized mass operator of the DC model is mapped into the mass operator of the Thirring model. The complete bosonization of the model is performed by computing the composite operators in the bosonized quantum Hamiltonian as the leading operators in the Wilson short distance expansions [11].

Notwithstanding the results on the weak equivalence between the DC model and the Thirring model, from our point of view, several algebraic and structural aspects of this equivalence, as proposed in [6–8], still remain obscure and the underneath property of the DC model which enables the claimed correspondence has never been clearly displayed within the operator formulation. A demonstration at the operator level that exhibits compactly the Thirring interaction behind the DC model has never been furnished and a clear understanding of this weak equivalence within the operator formulation still remains lacking in the literature. One of the purposes of this work is to fill this gap.

In this paper, we shall generalize the presentation of [11] and re-analyse the DC model using the operator approach. The DC model corresponds to the generalization of the model considered in [11] including a new degree of freedom of the massless scalar field interacting with the massive Fermi field via vector-current–scalar-derivative interaction. We show that the DC model is equivalent to the massive Thirring model with two vector-current-derivative couplings (Schroer–Thirring model). *The equivalence between the DC model and the Schroer–Thirring model is established at the operator level without imposing conditions neither on the nature of the bosonic fields, nor on the coupling parameters*. Following a different approach to those employed in [7, 8], the hidden Thirring interaction in the DC model is displayed by performing a canonical transformation on the Bose field algebra. The operator solution for the quantum equations of motion of the DC model corresponds to the Mandelstam Fermi field operator of the Thirring model interacting with two free bosonic scalar fields via vector-current-derivative couplings. The Thirring interaction is not affected by the introduction of the vector-current–scalar-derivative coupling corresponding to the Schroer model [1]. The Thirring interaction is an intrinsic property of the massless RS model [11]. The limit of the zero coupling parameter is well defined and the Schroer model [1], as well as the Rothe–Stamatescu model [2], is correctly recovered. The weak equivalence between the fermionic sector of the DC model and the Thirring model is established for certain values of the coupling parameters. Within the present approach the weak equivalence between the two models is exhibited compactly without artificially reducing the number of bosonic degrees of freedom.

The paper is organized as follows. In section 2 we present the operator formulation for the DC model and the equivalence with the *Schroer–Thirring model* is established. In section 3, we discuss the weak equivalence between the DC model and the massive Thirring model. Contrary to what is done in [7], the correspondence between the fermionic Wightman functions of the two models is established without reducing the number of degrees of freedom and is consistent with the introduction of the mass term for the Fermi field. In section 4 we discuss the DC model with massless fermions and its connection with the massless Rothe–Stamatescu model with a Thirring interaction (*massless Rothe–Stamatescu–Thirring model*). The conclusion is presented in section 5.

2. Operator solution in terms of the Thirring field

The classical Lagrangian density defining the two-dimensional derivative-coupling model of a massive Fermi field interacting with two massless Bose fields is given by [6–8][1]

$$\begin{aligned}\mathcal{L}(x) = \bar{\psi}(x)(i\gamma^{\mu}\partial_{\mu} - m_o)\psi(x) + \tfrac{1}{2}\partial_{\mu}\eta(x)\partial^{\mu}\eta(x) + \tfrac{1}{2}\partial_{\mu}\widetilde{\phi}(x)\partial^{\mu}\widetilde{\phi}(x) \\ + g(\bar{\psi}(x)\gamma^{\mu}\psi(x))\partial_{\mu}\eta(x) + \tilde{g}(\bar{\psi}(x)\gamma^{\mu}\gamma^{5}\psi(x))\partial_{\mu}\widetilde{\phi}(x),\end{aligned} \tag{2.1}$$

where $\eta(x)$ is a *scalar* field and $\widetilde{\phi}(x)$ is a *pseudoscalar* field. Except by the presence of a decoupled massless boson field, for $g = 0$ the model corresponds to the Rothe–Stamatescu (RS) model [2] in the zero mass limit of the pseudoscalar field $\widetilde{\phi}$ and modified to include a mass term for the fermion field (MRS model [11]), and for $\tilde{g} = 0$ it corresponds to the Schroer

[1] The conventions used are

$$\gamma^{0} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \gamma^{1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \gamma^{5} = \gamma^{0}\gamma^{1}, \epsilon^{01} = 1, \gamma^{\mu}\gamma^{5} = \epsilon^{\mu\nu}\gamma_{\nu}.$$

$$g^{00} = -g^{11} = 1\epsilon_{01} = \epsilon^{10} = 1$$

model [1]. The equations of motion defining the quantum theory are

$$(i\gamma^\mu\partial_\mu - m_o)\psi(x) = gN[\gamma^\mu\psi(x)\partial_\mu\eta(x)] + \tilde{g}N[\gamma^\mu\gamma^5\psi(x)\partial_\mu\widetilde{\phi}(x)], \tag{2.2}$$

$$\Box\eta(x) = -g\partial_\mu \vdots(\bar{\psi}(x)\gamma^\mu\psi(x))\vdots = 0, \tag{2.3}$$

$$\Box\widetilde{\phi}(x) = -\tilde{g}\partial_\mu \vdots(\bar{\psi}(x)\gamma^\mu\gamma^5\psi(x))\vdots. \tag{2.4}$$

The notation $\vdots(\bullet)\vdots$ in equations (2.3) and (2.4) means that the current is computed as the leading term in the Wilson short distance expansion [15] and the normal products in (2.2) are defined by the symmetric limit [2, 14]

$$N[\psi(x)\partial_\mu\Phi(x)] \doteq \lim_{\varepsilon\to 0}\tfrac{1}{2}\{\partial_\mu\Phi(x+\varepsilon)\psi(x) + \partial_\mu\Phi(x-\varepsilon)\psi(x)\}. \tag{2.5}$$

Due to the conservation of the vector current in equation (2.3) the scalar field η is free and massless. As a consequence of the axial-current-pseudoscalar derivative interaction, for massive fermions ($m_o \neq 0$) the pseudoscalar field $\widetilde{\phi}$ does not remain free due to the non-conservation of the axial current in equation (2.4),

$$\Box\,\widetilde{\phi}(x) = \mathrm{i}\tilde{g}m_o \vdots(\bar{\psi}(x)\gamma^5\psi(x))\vdots. \tag{2.6}$$

For massless Fermi fields the model described by the Lagrangian (2.1) is a scale invariant theory with anomalous scaling dimension [2]. As in the standard Thirring model [12], in order for the theory described by the Lagrangian (2.1) to have the model with a massless fermion as the short distance fixed point, the scale dimension of the mass operator must be

$$D_{\bar{\psi}\psi} < 2. \tag{2.7}$$

In what follows the mass term should be understood as a perturbation in the scale invariant model [11].

The operator solution for the quantum equations of motion is given in terms of Wick-ordered exponentials [2, 7, 14],

$$\psi(x) = \mathcal{Z}_\psi^{-\frac{1}{2}} : \mathrm{e}^{\mathrm{i}[g\eta(x)+\tilde{g}\gamma^5\widetilde{\phi}(x)]} : \psi^{(0)}(x), \tag{2.8}$$

where $\mathcal{Z}_\psi$ is a wavefunction renormalization constant [2, 14] and $\psi^{(0)}(x)$ is the free massive Fermi field,

$$(\mathrm{i}\gamma^\mu\partial_\mu - m_o)\psi^{(0)}(x) = 0. \tag{2.9}$$

The bosonized expression for the free massive Fermi field is given by the Mandelstam field operator,

$$\psi^{(0)}(x) = \left(\frac{\mu}{2\pi}\right)^{1/2} \mathrm{e}^{-\mathrm{i}\frac{\pi}{4}\gamma^5} : \exp\left(\mathrm{i}\sqrt{\pi}\left\{\gamma^5\widetilde{\varphi}(x) + \int_{x^1}^{\infty} \partial_0\widetilde{\varphi}(x^0, z^1)\,\mathrm{d}z^1\right\}\right):, \tag{2.10}$$

where μ is an infrared regulator reminiscent of the free massless theory. For $m_o = 0$, use can be made of the fact that

$$\epsilon_{\mu\nu}\partial^\nu\widetilde{\varphi} = \partial_\mu\varphi. \tag{2.11}$$

The meaning of the notation $:(\bullet):$ in the field operators is that the Wick ordering is performed by a point-splitting limit in which the singularities subtracted are of the free theory. In this way, the Wilson short distance expansions are performed using the two-point function of the free massless scalar field [14]

$$[\Phi^{(+)}(x), \Phi^{(-)}(0)]_{x\approx 0} = -\frac{1}{4\pi}\ln\{-\mu^2(x^2 + \mathrm{i}\epsilon x^0)\}. \tag{2.12}$$

We shall ignore the infrared problems of the two-dimensional massless free scalar field since the selection rules carried by the Wick-ordered exponentials ensure the construction of a positive metric Hilbert subspace [12, 13] for the fermionic sector of the model.

The vector current is computed with the regularized point-splitting limit procedure

$$J^\mu(x) = \lim_{\varepsilon\to 0} f(\varepsilon)\left\{\bar\psi(x+\varepsilon)\gamma^\mu \times \exp\left(-\mathrm{i}\int_x^{x+\varepsilon}[g\gamma^5\epsilon_{\mu\nu}\partial^\nu\eta(z)+\tilde g\epsilon_{\mu\nu}\partial^\nu\widetilde\phi(z)]\,\mathrm{d}z^\mu\right)\psi(x) - \mathrm{V.E.V.}\right\}, \tag{2.13}$$

with the wavefunction renormalization constant given by

$$\mathcal{Z}_\psi(\epsilon) = \exp(\tilde g^2[\widetilde\phi^{(+)}(x+\epsilon),\widetilde\phi^{(-)}(x)] + g^2[\eta^{(+)}(x+\epsilon),\eta^{(-)}(x)]), \tag{2.14}$$

and $f(\varepsilon)$ is a suitable renormalization constant. The vector current is given by

$$J^\mu(x) = j_f^\mu(x) - \frac{g}{\pi}\partial^\mu\eta(x) - \frac{\tilde g}{\pi}\epsilon^{\mu\nu}\partial_\nu\widetilde\phi(x), \tag{2.15}$$

where $j_f^\mu(x)$ is the free fermion current,

$$j_f^\mu(x) = -\frac{1}{\sqrt\pi}\epsilon^{\mu\nu}\partial_\nu\widetilde\varphi(x), \tag{2.16}$$

and the axial current is

$$J_\mu^5(x) = \epsilon_{\mu\nu}J^\nu(x) = -\frac{1}{\sqrt\pi}\partial_\mu\widetilde\varphi(x) - \frac{g}{\pi}\epsilon_{\mu\nu}\partial^\nu\eta(x) - \frac{\tilde g}{\pi}\partial_\mu\widetilde\phi(x). \tag{2.17}$$

The bosonized form of the quantum equations of motion (2.3) and (2.4) are

$$\left(1-\frac{g^2}{\pi}\right)\Box\,\eta(x) = 0, \tag{2.18}$$

$$\left(1-\frac{\tilde g^2}{\pi}\right)\Box\,\widetilde\phi(x) = \frac{\tilde g}{\sqrt\pi}\Box\,\widetilde\varphi(x). \tag{2.19}$$

The bosonized mass operator takes the form

$$\vdots(\bar\psi(x)\psi(x))\vdots = -\frac{\mu}{\pi}:\cos(2\sqrt\pi\widetilde\varphi(x)+2\tilde g\widetilde\phi(x)):, \tag{2.20}$$

and the γ^5-invariance breaking term arising from the fermion mass is given by

$$\vdots(\bar\psi(x)\gamma^5\psi(x))\vdots = i\frac{\mu}{\pi}:\sin(2\sqrt\pi\widetilde\varphi(x)+2\tilde g\widetilde\phi(x)):. \tag{2.21}$$

For $m_o = 0$, the axial current is conserved. In this case the pseudoscalar fields $\widetilde\varphi$ and $\widetilde\phi$ are both free and massless. Note that the scalar field η is a free massless field even for $m_o \neq 0$. From the bosonized mass operator (2.20) and from the equation of motion (2.19) we see that for $m_o \neq 0$ the fields $\widetilde\varphi$ and $\widetilde\phi$ are sine-Gordon-like fields. The mass operator is independent of the scalar (free) field η associated with the coupling parameter g of the vector-current–scalar-derivative interaction (Schroer model) in the Lagrangian (2.1). In this way the mass operator trivially commutes with the charge $\mathcal{Q}_\eta$ defined by

$$\mathcal{Q}_\eta = -\frac{g}{\pi}\int\partial_0\eta(x)\,\mathrm{d}x^1. \tag{2.22}$$

In order to have canonical commutation relations for the fields $\widetilde{\phi}$ and η we perform the field scaling

$$\widetilde{\phi}(x) = \left(1 - \frac{\tilde{g}^2}{\pi}\right)^{-\frac{1}{2}} \widetilde{\phi}'(x), \tag{2.23}$$

$$\eta(x) = \left(1 - \frac{g^2}{\pi}\right)^{-\frac{1}{2}} \eta'(x), \tag{2.24}$$

with $g^2 < \pi$ and $\tilde{g}^2 < \pi$ [11]. The mass operator (2.20), the vector current (2.15) and the equations of motion (2.18)–(2.19) can be rewritten as

$$\vdots(\bar{\psi}(x)\psi(x))\vdots = -\frac{\mu}{\pi} : \cos\left(2\sqrt{\pi}\widetilde{\varphi}(x) + \frac{2\tilde{g}}{\sqrt{1 - \frac{\tilde{g}^2}{\pi}}}\widetilde{\phi}'(x)\right):, \tag{2.25}$$

$$J_\mu(x) = -\frac{g}{\pi}\frac{1}{\sqrt{1 - \frac{g^2}{\pi}}}\partial_\mu \eta(x) - \frac{1}{\pi}\epsilon_{\mu\nu}\partial^\nu\left(\sqrt{\pi}\widetilde{\varphi}(x) + \frac{\tilde{g}}{\sqrt{1 - \frac{\tilde{g}^2}{\pi}}}\widetilde{\phi}'(x)\right), \tag{2.26}$$

$$\Box\eta'(x) = 0, \tag{2.27}$$

$$\Box\left(\sqrt{\pi}\widetilde{\phi}'(x) - \frac{\tilde{g}}{\sqrt{1 - \frac{\tilde{g}^2}{\pi}}}\widetilde{\varphi}(x)\right) = 0. \tag{2.28}$$

The scaling dimension of the mass operator is given by

$$D_{\bar{\psi}\psi} = \frac{\widetilde{\beta}^2}{4\pi}, \tag{2.29}$$

with $\widetilde{\beta}^2$ defined by

$$\widetilde{\beta}^2 \doteq \frac{4\pi}{1 - \frac{\tilde{g}^2}{\pi}}. \tag{2.30}$$

Consistence with (2.7) also requires that $\tilde{g}^2 < \pi/2$. The scaling dimension of the mass operator is the same as that of the massive Thirring model with the coupling parameter $\tilde{g}^2$. As shown in [11], this is a consequence of the fact that the existence of the Thirring interaction is an intrinsic property of the MRS model. In terms of the rescaled fields $\tilde{\phi}'$ and η', the Fermi field operator can be rewritten as

$$\psi(x) = \mathcal{Z}^{-1/2} : \exp\left(\mathrm{i}\left\{\frac{g}{\sqrt{1 - \frac{g^2}{\pi}}}\eta'(x) + \frac{\tilde{g}}{\sqrt{1 - \frac{\tilde{g}^2}{\pi}}}\gamma^5\widetilde{\phi}'(x)\right\}\right) : \psi^{(0)}(x). \tag{2.31}$$

For $m_o \neq 0$, the fields $\widetilde{\varphi}$ and $\widetilde{\phi}$ are not free fields, whereas the field η is a massless free field. From the bosonized expression for the mass operator (2.25) one sees that the sine-Gordon soliton field should be a combination of the fields $\widetilde{\varphi}$ and $\widetilde{\phi}$. In this way, on account of the combinations between the pseudoscalar fields $\widetilde{\varphi}$ and $\widetilde{\phi}'$ appearing in equations (2.25), (2.26) and (2.28), following the procedure introduced in [11], let us perform the canonical field transformation,

$$\tilde{\delta}\widetilde{\Phi}(x) = \sqrt{\pi}\widetilde{\varphi}(x) + \frac{\tilde{g}}{\sqrt{1-\frac{\tilde{g}^2}{\pi}}}\widetilde{\phi}'(x), \tag{2.32}$$

$$\tilde{\delta}\widetilde{\xi}(x) = \frac{\tilde{g}}{\sqrt{1-\frac{\tilde{g}^2}{\pi}}}\widetilde{\varphi}(x) - \sqrt{\pi}\widetilde{\phi}'(x). \tag{2.33}$$

Imposing canonical commutation relations for the fields $\widetilde{\Phi}$ and $\widetilde{\xi}$, the parameter $\tilde{\delta}$ is fixed as being

$$\tilde{\delta}^2 = \frac{\tilde{\beta}^2}{4} = \frac{\pi}{1-\frac{\tilde{g}^2}{\pi}}. \tag{2.34}$$

The field transformation (2.32)–(2.33) is consistent with the introduction of the mass perturbation since it does not mix the field $\widetilde{\eta}$ with the fields $\widetilde{\varphi}$ and $\widetilde{\phi}$. The fields $(\widetilde{\phi}', \widetilde{\varphi})$ can be written in terms of the new fields $(\widetilde{\phi}, \widetilde{\xi})$ as

$$\widetilde{\phi}'(x) = \frac{\tilde{g}}{\sqrt{\pi}}\widetilde{\Phi}(x) - \frac{\sqrt{\pi}}{\tilde{\delta}}\widetilde{\xi}(x), \tag{2.35}$$

$$\widetilde{\varphi}(x) = \frac{\sqrt{\pi}}{\tilde{\delta}}\widetilde{\Phi}(x) + \frac{\tilde{g}}{\sqrt{\pi}}\widetilde{\xi}(x). \tag{2.36}$$

The equation of motion (2.28) is then reduced to

$$\Box\widetilde{\xi}(x) = 0. \tag{2.37}$$

The vector current (2.26) can be rewritten as

$$J_\mu(x) = \eta_\mu(x) + \mathcal{J}_\mu^{\mathrm{Th}}(x), \tag{2.38}$$

where

$$\eta_\mu(x) = -\frac{g}{\pi}\frac{1}{\sqrt{1-\frac{g^2}{\pi}}}\partial_\mu\eta(x), \tag{2.39}$$

and the Thirring current is given by

$$\mathcal{J}_\mu^{\mathrm{Th}}(x) = -\frac{\widetilde{\beta}}{2\pi}\epsilon_{\mu\nu}\partial^\nu\widetilde{\Phi}(x). \tag{2.40}$$

As in the case of the MRS model [11], the field ξ does not contribute to the fermionic current. Using the fact that the field ξ is free and massless

$$\varepsilon_{\mu\nu}\partial^\nu\widetilde{\xi}(x) = \partial_\mu\xi(x), \tag{2.41}$$

and using (2.35)–(2.36), the Fermi field (2.8) can be rewritten as

$$\psi(x) = \mathcal{Z}_\psi^{-\frac{1}{2}} : \exp\left(\mathrm{i}\left[\frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(x) + \tilde{g}\xi(x)\right]\right) : \Psi(x), \tag{2.42}$$

where Ψ is the Fermi field operator of the massive Thirring model given by the Mandelstam operator [16]

$$\Psi(x) = \left(\frac{\mu}{2\pi}\right)^{1/2} \mathrm{e}^{-\mathrm{i}\frac{\pi}{4}\gamma^5} : \exp\left(\mathrm{i}\left\{\gamma^5\frac{\widetilde{\beta}}{2}\widetilde{\Phi}(x) + 2\pi\widetilde{\beta}^{-1}\int_{x^1}^{\infty}\partial_0\widetilde{\Phi}(x^0, z^1)\,\mathrm{d}z^1\right\}\right) :. \tag{2.43}$$

The bosonized mass operator (2.25) is now given by

$$\vdots(\bar{\psi}(x)\psi(x))\vdots \equiv \vdots(\bar{\Psi}(x)\Psi(x))\vdots = -\frac{\mu}{\pi} : \cos[\widetilde{\beta}\widetilde{\Phi}(x)]:. \tag{2.44}$$

The Thirring interaction is not affected by the introduction of the vector-current–scalar-derivative coupling corresponding to the Schroer model (g-coupling), which implies that the Thirring interaction is an intrinsic property of the massless RS model. The equation of motion (2.2) for the Fermi field can be rewritten as

$$(\mathrm{i}\gamma^\mu\partial_\mu - m_o)\psi(x) = \tilde{g}^2 N\big[\gamma^\mu\psi(x)\mathcal{J}_\mu^{\mathrm{Th}}(x)\big] + N[\gamma^\mu\psi(x)\{\tilde{g}\partial_\mu\xi(x) + g\partial_\mu\eta(x)\}]. \tag{2.45}$$

Equation (2.45) is the equation of motion for the massive Thirring model with two vector-current–scalar-derivative couplings, i.e., the Schroer–Thirring model [11].

The Wightman functions of the Fermi field operator $\psi(x)$ are those of the Fermi field $\Psi(x)$ of the Thirring model clouded by the contributions of the free fields η' and ξ,

$$\begin{aligned}\langle 0|\psi(x_1)\cdots\psi(x_n)\bar{\psi}(y_1)\cdots\bar{\psi}(y_n)|0\rangle &= \mathcal{W}(x_1,\ldots,x_n,y_1,\ldots,y_n)\\ &\quad\times\langle 0|\Psi(x_1)\cdots\Psi(x_n)\overline{\Psi}(y_1)\cdots\overline{\Psi}(y_n)|0\rangle,\end{aligned} \tag{2.46}$$

where

$$\begin{aligned}\mathcal{W}(x_1,\ldots,x_n,y_1,\ldots,y_n) &= \langle 0|\prod_{j=1}^{n} :\exp\left(\mathrm{i}\left[\frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(x_j) + \tilde{g}\xi(x_j)\right]\right)\\ &\quad\times :\prod_{k=1}^{n} :\exp\left(-\mathrm{i}\left[\frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(y_k) + \tilde{g}\xi(y_k)\right]\right) : |0\rangle.\end{aligned} \tag{2.47}$$

For $g = 0$, we obtain from (2.46) the Wightman functions of the Fermi field of the MRS model [11] and for $\tilde{g} = 0$ ($\widetilde{\beta}^2 = 4\pi$) the Wightman functions of the Schroer model [1] are recovered.

3. Weak equivalence of the DC model with the Thirring model

On account of equation (2.46), a one-to-one mapping between the fermionic Wightman functions of the DC model and those of the massive Thirring model can be established by imposing that the Wick-ordered exponential of the free Bose field combination,

$$\Omega(x) = g\eta(x) + \tilde{g}\xi(x), \tag{3.1}$$

generates infinitely delocalized states,

$$\mathcal{W}(x_1,\ldots,x_n,y_1,\ldots,y_n) = 1. \tag{3.2}$$

The set of fields $\{\tilde{\phi},\eta,\psi\}$ constitutes the intrinsic mathematical structure of the model and generates the intrinsic local field algebra $\Im$ defining the model. The metric quantization for these fields must emerge as a consequence of a structural algebraic condition intrinsic to the model. One may by a 'tour de force' obtain the weak equivalence between the DC model and the Thirring model by using the artifice of impose opposite metric quantization for the fields η and ξ. Of course, this means that we start from the beginning with the kinetic term for the field η in the Lagrangian (2.1) with a minus sign. In this case the equation of motion (2.18) is replaced by

$$\left(-1 - \frac{g^2}{\pi}\right)\Box\,\eta(x) = 0. \tag{3.3}$$

The canonical field η' is obtained after performing the field scaling

$$\eta'(x) = \left(1 + \frac{g^2}{\pi}\right)^{1/2} \eta(x), \tag{3.4}$$

such that

$$\langle 0|\eta'(x)\eta'(y)|0\rangle = -\langle 0|\xi(x)\xi(y)|0\rangle. \tag{3.5}$$

The Fermi field operator (2.42) can be rewritten as

$$\psi(x) = \omega(x)\Psi(x), \tag{3.6}$$

where

$$\omega(x) =: \mathrm{e}^{\mathrm{i}\Omega(x)} :=: \exp\left(\mathrm{i}\left\{\tilde{g}\xi(x) + \frac{g}{\sqrt{1+\frac{g^2}{\pi}}}\eta(x)\right\}\right):. \tag{3.7}$$

The operator $\Omega(x)$ would commute with itself,

$$[\Omega(x), \Omega(y)] = 0, \ \forall(x, y), \tag{3.8}$$

provided that the coupling parameters g and $\tilde{g}$ are not independent and being related by

$$\left(1 - \frac{\tilde{g}^2}{\pi}\right)\left(1 + \frac{g^2}{\pi}\right) = 1. \tag{3.9}$$

Relation (3.9) between the coupling parameters is similar to one of the relations obtained in [7][2]. Condition (3.9) implies that

$$\frac{g^2}{\pi} = \frac{\tilde{\beta}^2}{4\pi} - 1. \tag{3.10}$$

Under these assumptions the exponential operator $\omega(x)$ by itself generates constant Wightman functions. Since the operator $\omega(x)$ commutes with itself and also commutes with the Thirring field Ψ, the general Wightman functions generated by the Fermi field (3.6) are isomorphic to those generated by the Thirring field.

The Hilbert space of the standard DC model, as defined from the Lagrangian (2.1), is a representation of the local field algebra $\Im$, generated by the intrinsic fields $\{\widetilde{\phi}, \eta, \bar{\psi}, \psi\} \equiv \{\widetilde{\Phi}, \eta, \xi\}$,

$$\mathcal{H} = \Im|0\rangle. \tag{3.11}$$

In the modified model, in which we assign negative metric quantization for the field η, the Hilbert space of states is a representation of the field algebra $\Im'$, i.e., $\mathcal{H}' = \Im'|0\rangle$. The operator $\omega(x)$ does not commute with the vector current $J^\mu \in \Im'$, given by (2.38), and thus carries the charge $\mathcal{Q}_\eta$. This implies that *the operator* $\omega(x)$ *does not reduce to the identity in the Hilbert space* $\mathcal{H}'$. It is the identity operator only in a proper subspace of states $\mathcal{H}'_{\mathrm{Th}} \subset \mathcal{H}'$ defined by the set of Wightman functions of the Fermi field operator ψ. In the Hilbert subspace $\mathcal{H}'_{\mathrm{Th}}$, the position independence of the operator $\omega(x)$ can be expressed in the weak form as

$$\begin{aligned}\langle 0|\omega(x_1)\cdots\omega(x_\ell)\omega(x'_1)\cdots\omega(x'_\ell)\psi(y_1)\cdots\psi(y_n)\bar{\psi}(z_1)\cdots\bar{\psi}(z_n)|0\rangle \\ = \langle 0|\Psi(y_1)\cdots\Psi(y_n)\bar{\Psi}(z_1)\cdots\bar{\Psi}(z_n)|0\rangle.\end{aligned} \tag{3.12}$$

[2] A similar expression for the Fermi field in terms of the Mandelstam operator was obtained in [7]. In this case the 'spurion' field σ that generates constant Wightman functions is defined in terms of a combination of the three fields $\widetilde{\varphi}, \widetilde{\phi}$ and $\widetilde{\eta}$. However, since in the model with massive fermions the fields $\widetilde{\varphi}$ and $\widetilde{\phi}$ are sine-Gordon fields and $\widetilde{\eta}$ remains free and massless, the field redefinition used in [7] becomes meaningless. The coupling parameter relation (3.9) is obtained in [7] with the field $\widetilde{\phi}$ quantized with negative metric and for $\widetilde{\beta}^2 < 4\pi$.

Since the Bose fields η and ξ belong to the field algebra $\Im'$, one can define in $\mathcal{H}'$ the Thirring field,

$$\Psi(x) = \psi(x)\omega^{-1}(x), \tag{3.13}$$

and the Thirring current,

$$\mathcal{J}^{\mu}_{\mathrm{Th}}(x) = J^{\mu}(x) - \eta^{\mu}(x), \tag{3.14}$$

such that the Hilbert subspace $\mathcal{H}'_{\mathrm{Th}}$ is generated from the field subalgebra $\Im'_{\mathrm{Th}}\{\Psi, \mathcal{J}^{\mu}_{\mathrm{Th}}\} \subset \Im'\{\psi, \eta, \xi\}$. In the standard DC model, as defined from the Lagrangian (2.1), the weak equivalence with the Thirring model cannot be established in terms of the field operators defining the intrinsic local field algebra $\Im$.

4. DC model with massless fermions: equivalence with the massless Rothe–Stamatescu–Thirring model

In this section we shall consider the connection between the DC model with massless fermions and the massless Thirring model with the coupling parameter g. To this end we shall consider an alternative approach by introducing a canonical field transformation which only has meaning for the DC model with massless fermions. To begin with, let us consider the operator solution (2.8) for $m_o = 0$. Since in this case the field $\widetilde{\varphi}$ is free and massless, using that

$$\varphi(x) = \int_{x^1}^{\infty} \partial_0 \widetilde{\varphi}(x^0, z^1)\, \mathrm{d}z^1, \tag{4.1}$$

we can write the Fermi field as

$$\psi(x) = \left(\frac{\mu}{2\pi}\right)^{1/2} : \exp\left(\mathrm{i}\left[\frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(x) + \gamma^5 \frac{\tilde{g}}{\sqrt{1-\frac{\tilde{g}^2}{\pi}}}\widetilde{\phi}'(x)\right]\right)$$
$$\times :: \exp(\mathrm{i}\sqrt{\pi}[\gamma^5\widetilde{\varphi}(x) + \varphi(x)]):, \tag{4.2}$$

and the vector current (2.15) can be rewritten as

$$J_{\mu}(x) = \phi_{\mu}(x) - \frac{1}{\pi}\partial_{\mu}\left(\sqrt{\pi}\varphi(x) + \frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(x)\right), \tag{4.3}$$

where

$$\phi_{\mu}(x) = -\frac{1}{\pi}\frac{\tilde{g}}{\sqrt{1-\frac{\tilde{g}^2}{\pi}}}\epsilon_{\mu\nu}\partial^{\nu}\widetilde{\phi}'(x). \tag{4.4}$$

Let us now perform a canonical transformation on the free massless scalar fields η' and φ depending on the coupling parameter g,

$$\delta\Sigma(x) = \sqrt{\pi}\varphi(x) + \frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\eta'(x), \tag{4.5}$$

$$\delta\zeta(x) = \frac{g}{\sqrt{1-\frac{g^2}{\pi}}}\varphi(x) - \sqrt{\pi}\eta'(x), \tag{4.6}$$

with

$$\delta^2 = \frac{\pi}{1-\frac{g^2}{\pi}}. \tag{4.7}$$

The Fermi field operator (4.2) can be rewritten as

$$\psi(x) = :\exp\left(\mathrm{i}\gamma^5\left[g\widetilde{\zeta}(x) + \frac{\tilde{g}}{\sqrt{1-\frac{\tilde{g}^2}{\pi}}}\widetilde{\phi}'(x)\right]\right):\Psi(x), \tag{4.8}$$

where $\Psi(x)$ is the Fermi field operator of the massless Thirring model

$$\Psi(x) =: \exp\left(\mathrm{i}\frac{\beta}{2}\gamma^5\tilde{\Sigma}(x) + \mathrm{i}\frac{2\pi}{\beta}\Sigma(x)\right): \tag{4.9}$$

with

$$\frac{4\pi}{\beta^2} = \frac{1}{1-\frac{g^2}{\pi}}. \tag{4.10}$$

The vector current (4.3) is given by

$$J_\mu(x) = \phi_\mu(x) + J_\mu^{\mathrm{Th}}(x), \tag{4.11}$$

where the Thirring current is

$$J_\mu^{\mathrm{Th}}(x) = -\frac{2}{\beta}\epsilon_{\mu\nu}\partial^\nu\tilde{\Sigma}(x). \tag{4.12}$$

The field $\zeta(x)$ does not contribute to the fermionic current. For $\tilde{g} = 0$, the Fermi field (4.8) corresponds to the operator solution of the massless Rothe–Stamatescu model with a Thirring interaction (*massless Rothe–Stamatescu–Thirring model*). The equation of motion (2.2) is now given by

$$\mathrm{i}\gamma^\mu\partial_\mu\psi(x) = g^2 N\left[\gamma^\mu\psi(x)J_\mu^{\mathrm{Th}}(x)\right] + N[\gamma^\mu\gamma^5\psi(x)\{g\partial_\mu\widetilde{\zeta}(x) + \tilde{g}\partial_\mu\widetilde{\phi}(x)\}]. \tag{4.13}$$

It should be stressed that the field transformation defined by equations (4.5)–(4.6) only can be implemented in the model with massless fermions, since in equation (4.3) use was made of the fact that the field φ is a free massless field. For $m_o = 0$, due to the conservation of both vector and axial currents, the fields η, φ and ϕ are free and massless. For massive fermions the field $\widetilde{\varphi}$ is a sine-Gordon-like field, whereas the field η remains free and massless and in this case the transformation (4.5)–(4.6) is meaningless since it mixes a free field with an interacting one. As a matter of fact, the transformation (4.5)–(4.6) spoils the mass operator. For massless fermions one can rewrite the transformation (4.5)–(4.6) in terms of the corresponding pseudoscalar fields and the mass operator can be written as

$$:\cos(2\sqrt{\pi}\widetilde{\varphi}(x) + 2\tilde{g}\widetilde{\phi}(x)) :=: \cos(\beta\widetilde{\Sigma}(x) + 2g\widetilde{\zeta}(x) + 2\tilde{g}\widetilde{\phi}(x)):. \tag{4.14}$$

This explains why the field redefinition introduced in [7] becomes meaningless for massive fermions.

4.1. *Weak equivalence with the massless Thirring model*

The weak equivalence with the massless Thirring model can be established by considering the field ϕ quantized with negative metric such that

$$\widetilde{\phi}' = \sqrt{1+\frac{\tilde{g}}{\pi}}\widetilde{\phi}. \tag{4.15}$$

The Fermi field operator is given in terms of Wick exponential of the field combination

$$\widetilde{\theta}(x) = g\tilde{\zeta}(x) + \frac{\tilde{g}}{\sqrt{1+\frac{\tilde{g}}{\pi}}}\widetilde{\phi}'(x) \tag{4.16}$$

as follows:

$$\psi(x) = :\mathrm{e}^{\mathrm{i}\gamma^5\widetilde{\theta}(x)}:\Psi(x). \tag{4.17}$$

The Wick-ordered exponential of the field $\widetilde{\theta}(x)$ generates constant Wightman functions provided that the coupling constants are related by

$$\left(1+\frac{\tilde{g}^2}{\pi}\right)\left(1-\frac{g^2}{\pi}\right) = 1, \tag{4.18}$$

implying that[3]

$$\frac{\tilde{g}^2}{\pi} = \frac{4\pi}{\beta^2} - 1. \tag{4.19}$$

In this case the Wightman functions of the Fermi field (4.8) are isomorphic to those of the massless Thirring model with the coupling constant g.

5. Conclusion

The presence of a hidden Thirring interaction in the DC model is an intrinsic property of the model independent of any relation between the coupling parameters g and $\tilde{g}$. The DC model is isomorphic to the Schroer model with a Thirring interaction (Schroer–Thirring model). The weak equivalence with the Thirring model can be established without reducing the number of degrees of freedom. This is achieved by starting from a modified DC model in which the bosonic fields enter with opposite metric. In this modified DC model the general Wightman functions of the Fermi field operator are isomorphic to those of the Thirring model. For massless fermions the DC model can be mapped into the massless Rothe–Stamatescu–Thirring model.

Acknowledgment

The authors are grateful to Brazilian Research Council (CNPq) for partial financial support.

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